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ON A THEOREM RELATING TO CLOSED PLANE CURVES.

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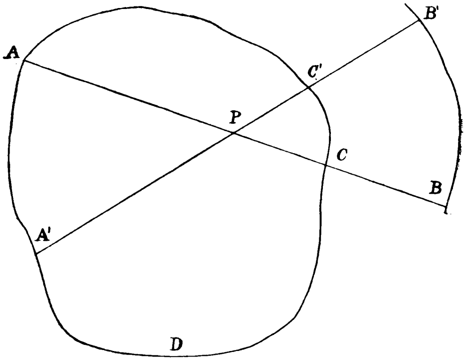
In Williamson's *Calculus* a proof of the following theorem is given: A chord of fixed length l slides round a closed plane curve, its extremities being constantly on the curve. A point on this chord, at a distance a from one end, and b from the other, describes a second closed curve. The area included between these curves is πab.

The following demonstration is interesting on account of a curious fact involved in it, and expressed by equation (2).

Let ACD be the given closed curve, AC the given chord, and B the given point on the chord. Let B' be the position of B when the chord AC has moved to its consecutive position $A'C'$, and let P be the point of intersection of AC and $A'C'$.



Let $AC = l$, $AB = a$, $CB = b$, $PC = x$, and $\angle B'PB = d\theta$. Let α be the area sought, and $BCC'B' = d\alpha$. Then

$$d\alpha = \tfrac{1}{2}(x + b)^2 d\theta - \tfrac{1}{2}x^2 d\theta = bx d\theta + \tfrac{1}{2}b^2 d\theta.$$

Integrating between 0 and 2π, $\alpha = b\int_0^{2\pi} x d\theta + \pi b^2$. (1)

We cannot evaluate the integral $\int_0^{2\pi} x d\theta$ by ordinary methods, because we do not know the relation between x and θ. We observe, however, that it is independent of b, and that if we can find its value for any particular value of b, the value so found will hold for any other value of b.

If we make $b = 0$, and $\therefore \alpha = 0$, (1) leads to the identity $0 = 0$. But putting $b = -l$ and noting that this makes B coincide with A, so that $\alpha = 0$, equation (1) gives us

$$0 = -l\int_0^{2\pi} x d\theta + \pi l^2, \quad \text{or} \quad \int_0^{2\pi} x d\theta = \pi l. \qquad (2)$$

Substituting this value of the integral in (1), we have

$$\alpha = \pi lb + \pi b^2 = \pi (l + b) b = \pi ab.$$